**Lesson 0**

Introduction to Complex Numbers

The set of all complex numbers is essentially a two-dimensional extension of the field of real numbers. By a complex number we mean a number comprising a real and an imaginary part.

It can be written in the form ;

where and are real numbers, and i is postulated to be the imaginary unit with the property

Complex numbers help in two areas:

- application in real-world applications

- simplifying mathematics

**Cartesian Form**

or

Where and are both real numbers and is known as the imaginary unit and satisfies .

The number is called “the real part of ”

The number is called “the imaginary part of ”

**Lesson 2**

Graphical Representation

<https://jutanium.github.io/ComplexNumberGrapher/>

Complex numbers are represented on the complex plane (Argand diagram) with:

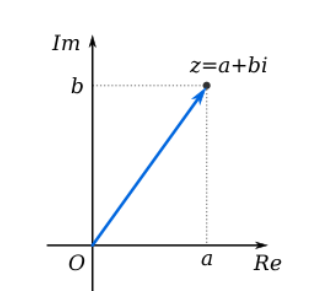
- a “real” (horizontal) axis

- an “imaginary” (vertical) axis

These are written in the form

The number represents a rotation from the real number line

Therefore, a complex number can represent a point, with the real part representing the position on the horizontal, real number line and the imaginary part representing the position on the imaginary or vertical axis.



Operations with complex numbers use the properties of to transform these points.

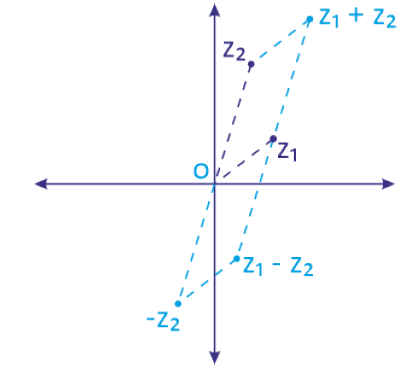
Example: Square the complex number

**Lesson 2**

Operations on complex numbers

**Addition/Subtraction**

*Just add/subtract the corresponding components*

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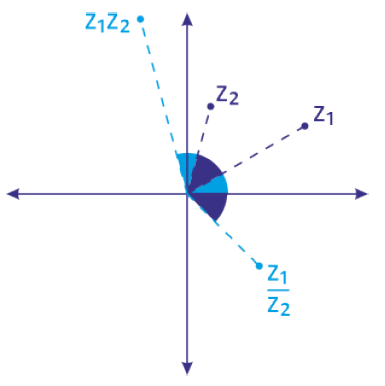
If and

Addition of two complex numbers can be represented as a parallelogram.



**Scalar Multiplication**

If

Scalar multiplication of complex numbers is represented through the stretching principle.

**Multiplication of two complex numbers**

If and

Multiplication of two complex numbers is done through rotation and stretching. The length of is the product of and . The angle of is the sum of and

*Lengths calculated using Pythagoras theorem.*

*Angles calculated using trig.*

**Division by a complex number**

*Multiply by a conjugate (turns denominator into real number)*

If and

*but remember*

**Quadratic formula**

**Lesson 2**

Polar Coordinates

<https://www.cimt.org.uk/projects/mepres/alevel/fpure_ch3.pdf>

Can be expressed as (3,4) on a argand diagram

modulus

Can be expressed as length & direction,

The above is written as:

*modulus of z*

The angle of is

argument

The above is written as

is the argument or phase of

Polar form is just a combination of the above

Example: Write in polar form

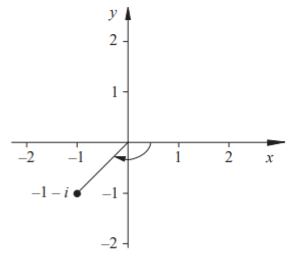
**Modulus**

[1] rewrite in the form

*already in the correct form*

[2] express as

*OA: a line from origin to point a*



**Argument**

[1] find the angle of in OA

*if a and b are positive*

Convert degrees to radians:

Principal value =

Therefore

**Lesson 3**

Sketching regions in the complex plane

<https://www.youtube.com/watch?v=8gtnZ5xSLuE>

**General case**

and

This represents a “wedge”

(could actually be a disk if and are large enough)

*is a complex number*

*is appositive real number*

*and are real numbers between and*

We want to include all the complex numbers that surround

We also want to rotate between and

*Assume is negative and is positive*

Region of interest: the region we will sketch is somewhere between and



<https://www.maths.usyd.edu.au/MATH1921/r/2018s1/pdf/tut02s.pdf>

<https://www.slader.com/discussion/question/sketch-the-following-sets-and-determine-which-are-domains-a/>

|  |  |  |  |
| --- | --- | --- | --- |
| **Form** | **Example** | **Example Set** | **Type of sketch** |
| and |  | and | “wedge” between and |
|  |  |  | Portion of the entire plane |
|  |  |  | Circle region.  Centered at of radius |
|  |  |  | Circle region. Centered at of radius |
|  |  |  | Region outside circle. Centered at of radius 2 |
|  |  |  | Region outside circle. |
|  |  |  | Region |
|  |  |  |  |
|  |  |  |  |

Example:

Sketch the region in the complex plane defied by all those complex numbers such that

and

For our problem:

tells us we don’t do any rotation

tells us we rotate radians



Don’t draw this line

Draw this line

Example: Sketch:

*Remember that is the same as*

<https://math.stackexchange.com/questions/782641/sketching-variations-of-argz>



For our problem:

tells us we don’t do any rotation

tells us we rotate radians, or an eighth of the plane

We also move the region one unit left

*s*

Example: Sketch:

*Remember that the modulus is smaller than the argument*

<https://www.doubtnut.com/question-answer/interpret-the-loci-arg-zpi-4-in-complex-plane-2109429>



For our problem:

tells us we don’t do any rotation

tells us we rotate radians, or an eighth of the plane

Example: Sketch:



**Lesson 4**

Exponential Form

and y

*euler’s formula*

*polar form in exponential form*

Example:

The complex number

It’s exponential form:

*also written as*

**Roots of complex numbers**

**Lesson 6**

Analytic Functions

**Cauchy-Rieman equations**

[1] Show that a function is differentiable everywhere

[2] Verify that the **Cauchy-Rieman equations** are satisfied everywhere

Example: ASS 1 2021. Let g be defined by

, where

- Find all points where g is differentiable.

- Is analytic at any point of ? Give reasons for your answer.

**Cauchy-Rieman equations**

[1] Show that a function is differentiable everywhere

Suppose that . The complex-valued function is differentiable at any point z in the complex plane:

The real part and the imaginary part are

And their partial derivatives are

All the partial derivatives are polynomial and thus continuous. Therefore, the function is differentiable wherever the Cauchy-Rieman equations are satisfied

[2] Verify that the Cauchy-Rieman equationsare satisfied everywhere

If the Cauchy–Riemann equations are to hold at a point (x, y), it follows that and , or that .

We have that:

holds

Thus, holds:

or

Therefore, the Cauchy-Rieman equationsare satisfied at the lines and .

[3] Verify if is analytic at any point of .

In order to be analytic, a function needs the partial derivatives of the real part and the imaginary part should:

- satisfy Cauchy-Rieman equations and , and

- be continuous

There is no neighborhood of any point throughout which is analytic as Cauchy-Rieman does not hold for an open set. Every neighborhood of any point will have points which are not on the lines and . Therefore, is nowhere analytic.

**Lesson 10**

Properties of z

**Modulus of z**

and

**Polar form**

,

*as would be undefined*

*cannot be negative, is the length of radius vector for*

; principal of

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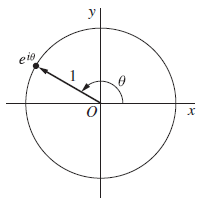
**Distance between two points**

*circle: centre*

**Exponential Form**

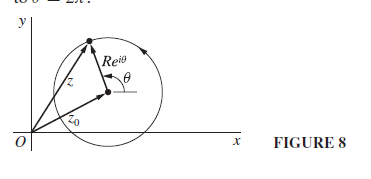
and y

*euler’s formula*

**

;

*polar form in exponential form*



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